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Financial Accounting Matrix and Transaction Matrices: A Concise Formalism for Describing Financial Stock Dynamics

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Abstract

Financial Accounting Matrix/Transaction Matrices is a novel way to describe financial stock dynamics, both for model construction and data analysis, that cleanly integrates with the Social Accounting Matrix formalism for describing financial flows.

Its purpose is to address the following issues: Firstly, a financial transaction typically affects several asset stocks, making it difficult to see which transactions gave rise to the observed behavior of a given stock. Secondly, every financial transaction is subject to net wealth constraints, and specifying and observing these can substantially increase the effort of model specification. Finally, while a Social Accounting Matrix is a powerful technique for tracing money flows in an economy, it is not very good at describing stock-flow relationships and stock-specific issues such as revaluation.

By arranging the financial stocks in a Financial Accounting Matrix (FAM) according to holder, issuer and denomination, and by using a collection of constant matrices (Transaction Matrices) that describe the structure of the financial sector in a given country, we can decompose the observed changes in the financial stocks into revaluation, net lending, and capital transactions in a computationally efficient way. Conversely, the FAM/TM formalism allows us to explicitly specify portfolio allocation behavior of the different institutions in an economy while automatically observing their net wealth constraints.

Thus, the proposed formalism cleanly separates universal accounting constraints, country-specific financial sector structure, and behavior of agents within that structure.

Keywords: Accounting, financial stock dynamics, Financial Accounting Matrix, portfolio balance

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1 Introduction

Financial stock dynamics are an essential part of any economic model or analysis that has a macro component. This applies to venerable classics such as the IS/LM model [Hicks 1937], as well as to more recent efforts. Sometimes, such as in Godley [1999], stock-flow relationships are at the core of the models. In other cases, such as in recent Computable General Equilibrium (CGE) models [Bourguignon et al. 1991, Rosensweig and Taylor 1990, Naastepad 2001], financial stock dynamics are grafted on top of an equilibrium framework because of the need to represent macroeconomic policies.

The financial stocks used typically include at least some definition of money supply and domestic government debt, but any halfway realistic representation of a financial system adds at least half a dozen others - foreign exchange reserves, demand deposits, private sector debt, etc. Each financial stock is a liability of some "institution" in the economy (such as the government, the central bank, or the "rest of the world") that is the issuer of the stock in question, and an asset of another such institution, the holder of that stock.

An important characteristic of financial stocks is that the composition of an institution's portfolio (i.e. the specific combination of its assets and liabilities) can change much faster than the same institution's net worth (the total current value of its assets minus the total current value of its liabilities). Therefore, any given financial transaction typically affects at least one asset and one liability stock of a given institution, subject to a net worth constraint.

The result of that is firstly, a proliferation of notations for financial stocks and secondly, a proliferation of implicit net worth constraints that all financial transactions must fulfill. Furthermore, the observed behavior of any given stock is typically influenced by several transactions, making attribution far from trivial.

These issues call, in our view, for a systematic, intuitive notation for all possible financial stocks in a given model, as well as for a method of describing financial stock behavior in a way that automatically observes net wealth constraints, and finally, for a method to disaggregate observed financial stock behavior into a simple vocabulary of atomic transactions. In short, we think there is a need for a concise formalism for describing financial stock dynamics, and the purpose of this article is to propose such a formalism.

We take our inspiration from a well-established formalism for describing nominal *flows* in an economy, namely Social Accounting Matrices. A Social Accounting Matrix is a matrix listing all nominal money flows in an economy, by source (column) and destination (row). This formalism has proven useful and intuitive enough that it is at present universally used to organize nominal flows. The Financial Accounting Matrix/Transaction Matrix formalism we are about to introduce is similar to the SAM formalism in that stocks are listed by holder and issuer. However, due to the specific properties of the financial stocks, it also has important dissimilarities.

In fact, at the same time as being partially inspired by the SAM, the FAM/TM formalism is also able to remedy what we perceive as a weakness of the SAM approach, namely its representation of stock-flow relationships. A SAM traditionally is assumed to have no leaks; that is, a well-formed SAM must "balance", meaning the sum of each row must equal the sum of the corresponding column. This is the same as saying that the total inflows into each account must equal total outflows, which in turn means there can be no accumulation of stocks anywhere. This abstraction can perhaps be justified if one works with only one time slice, as e.g. CGE models do, though even there stocks have been introduced in various ad hoc ways. The assumption of flow equilibrium, however, seems entirely untenable if one wants to consider a series of SAMs and financial stocks for several years. Each of the institutions will typically have a deficit or a surplus (government deficit, current account deficit, etc.) that will over time determine the behavior of important stocks such as the money supply or the foreign debt.

The question thus arises of how best to integrate financial stocks with the SAM formalism. We are certainly not the first to attempt that. In fact, the Social Accounting Matrix methodology has itself been adapted to describe stocks and stock-flow relationships. However, as the description of that technique in Taylor [1990] illustrates, the resulting SAMs become large and unwieldy very quickly, and still have problems dealing with revaluation of financial stocks. We propose another approach, namely combining an almost-unchanged SAM with a Financial Accounting Matrix. Briefly, we throw out the flow-of-funds accounts of the SAM and feed the net lending flows into the FAM instead. However, implementing this fairly intuitive idea turns out to be not quite trivial: if there are N different institutions in the model, there are N net lending flows but about N^2 possible FAM entries (issuer/holder combinations). Section 3.3 describes how that is addressed.

A final important difference between the SAM and the FAM is that while most processes in the SAM (such as e.g. private purchases of food) are described by a single entry, even "simple" financial transactions typically affect more than one entry of the FAM. For example, a bank loan to the private sector will in the simplest case increase the stock of loans of the private sector from the banks, as well as increase the stock of private sector's deposits by the same amount. This makes the task of disentangling the different causes of changes in financial stocks far from trivial. We will address that by augmenting the FAM with a formalism called Transaction Matrices, see Sections 3.4 and 4.

We begin our discussion with Section 2 that introduces and discusses the Financial Accounting Matrix notation. Then, Section 3 specifies a compact way of decomposing the changes in financial stocks into revaluation, capital transactions, and net lending; Section 4 explains how to operationalize this decomposition starting with real-world financial stock data; and Section 5 concludes.

2 The Financial Accounting Matrix

The first notation we would like to introduce is the Financial Accounting Matrix (FAM). The purpose of the Financial Accounting Matrix is keeping track of financial stocks in an economy, that is, of liabilities of different agents in an economy toward one another.

The idea of arranging financial stock data in a FAM is certainly not novel, and FAMs for different countries have been compiled since at least the 1960s [Goldsmith 1966]. However, it has remained surprisingly obscure, with most authors, especially in macro theory and modeling contexts, using sets of balance sheets instead. Therefore, before showing how we can take advantage of the FAM format

to develop a concise description of financial stock dynamics, let us briefly discuss the FAM concept itself.

In the same way that a Social Accounting Matrix (SAM) is a systematic listing of monetary flows in an economy, the Financial Accounting Matrix (FAM) is a systematic listing of the financial stocks, in a square table not dissimilar to a SAM.

In the same way that a flow of money always goes *from* an agent *to* another agent, a financial asset is a liability of one agent to another. In the FAM, assets of agents are arranged in rows, and liabilities in columns. Thus for example, the intersection of a row titled "Households" and a column titled "Commercial Banks" would be an asset of households and a liability of commercial banks, that is, deposits held by households at the commercial banks.

While the SAM and the FAM thus share some common features, there are also important differences between them. Possibly the most important one is that the FAM need not fulfill any "balancing" requirement. In fact, the difference between the total assets (row sum) and total liabilities (column sum) of an agent equals the agent's net worth, and can be positive or negative.

To date, the usual way of arranging financial asset and liability stocks for use in macroeconomic textbooks and models is balance sheets. The only instance known to the author of arranging financial stocks for use in a model in the form of a FAM, rather than a series of double-entry balance sheets, is Easterly [1990].

The difference between a FAM and a set of balance sheets very closely parallels the difference between a SAM and a T-shaped sources/uses table. While a sources/uses table only lists the income and expenditure flows of each institution i (leaving us very few clues as to which other institutions these flows are coming from/going to), a SAM allows us to directly see the flows from institution i to institution j, and we can obtain the sources/uses table from the SAM simply by aggregating over rows and columns. Further, a SAM automatically enforces some (though not all) accounting restrictions, namely that each flow has to go from one account to another, with no flows starting or ending in a void. By the simple act of placing a flow in a SAM cell, we ensure that it has an origin and a destination, and that over the whole economy, inflows equal outflows; while in a sources/uses table, this is not automatically observed and has to be imposed as an extra condition.

As we have just mentioned, the relationship of a FAM to a set of balance sheets is akin to the relationship of a SAM to a sources/uses table. In the double-entry balance sheet notation, any financial stock must appear twice, once as somebody's asset and another time as somebody else's liability. Thus there is twice the number of symbols to keep track of (not a negligible problem as there is typically a lot of symbols already), and the requirement "every asset appears twice" generates a handful of additional equations to keep track of – while with a FAM, it's automatically satisfied as a FAM is a single-entry system.

To see how this works, let us take an example of a set of balance sheets from recent literature and convert them into a FAM.

2.1 A Simple Example

The balance sheets we choose come from [Taylor 2004, Ch.10], reproduced in Table 1. We have here two countries, each divided into government, the banking system, and the nonbank private sector. Both governments issue bonds, both banking systems issue money backed by a mixture of domestic and foreign bonds, and both private sectors hold a mixture of both bonds and of money of their respective country, in addition to physical capital. With so many different stocks in play, the notation is of necessity somewhat involved, and rather than talk our way through it, we only remark that e is the exchange rate, various species of T refer to the different bonds, and the two Ks are physical capital stocks held by the respective private sectors. The balance sheets of each country are kept in that country's currency. Apart from a degree of symmetry between the two countries, little structure jumps to the untrained eye from Table 1.

Now let us see how that converts into a FAM in Table 2. We have arranged all asset stocks

Home Country				
Private Sector		Banking System		Government
M	Ω	T_b	M	T
T_b		eR^*		
eT_b^*				
qPK				
Foreign Country				
Private Sector		Banking System		Government
M^*	Ω^*	T_b^*	M^*	T^*
T_f^*		R/e		
T_f/e				
$q^*P^*K^*$				

Table 1: *The Balance Sheets in [Taylor 2004, Ch.10]*

Holder \ Issuer	Home Govt.	Foreign Govt.	Home Banks	Foreign Banks	*Total Financial Assets*
Home Banks	T_b	$\mathbf{eR}^*$			M
Foreign Banks	R	$\mathbf{eT_b^*}$			eM^*
Home Private	T_h	$\mathbf{eT_h^*}$	M		$\bar{\Omega}$
Foreign Private	T_f	$\mathbf{eT_f^*}$		$\mathbf{eM}^*$	$e\bar{\Omega}^*$
Total Liabilities	T	eT^*	M	eM^*	

Table 2: *The Balance Sheets of [Taylor 2004, Ch.10] Converted Into a Financial Accounting Matrix (Bold type denotes assets denominated in foreign currency)*

according to their respective holder/issuer, and added row and column totals. Since in Table 1 the governments have no financial assets and the households have no financial liabilities, we have omitted the corresponding rows/columns. All stocks have been converted into the currency of the home country.

The attentive reader will have noticed the conspicuous absence of the terms qPK and $q^*P^*K^*$ from the FAM. The reason for that is that they do not refer to $financial$ asset stocks, but to stocks of *physical* capital (be it inventories or equipment). As a result, they are an asset of the private sector, but not anybody's liability, and have to be accounted for separately. While the FAM does not thus include the physical capital stocks, it is very well capable of accounting for *equity*, that is, the value of the firms themselves, held e.g. by households, as equity is again a *financial* asset stock, a liability of the issuer and an asset of the holder. As physical capital is not part of the FAM, the rightmost column of Table 2 represents total *financial* assets of the respective institution; thus, $\bar{\Omega}$ and $e\bar{\Omega}^*$ represent the net financial worth of the respective private sector measured in home currency, as opposed to their total net worth Ω and $e\Omega^*$ of Table 1.

What does a glance at Table 2 tell us? Firstly, the meaning of various entries is almost completely determined by their positions (apart from denomination, which we'll consider a bit later). Thus, we

don't have to remember what variable R referred to - from the table we see that R is a liability of the home government and an asset of the foreign banking system, thus denoting home bonds held as forex reserves by the foreign banking system. Also, we see immediately that T_b, R, T_h, and T_f all refer to home bonds held by different institutions, and that T is their sum - something not at all obvious from Table 1, at least to the author.

Another advantage of the FAM format is that some accounting constraints that had to be derived with some effort from the set of balance sheets, are obvious from just staring at the FAM. For example, the sum of the row totals clearly must equal the sum of column totals (both being equal to the sum of all the FAM entries); applying that to our FAM, we see at once that $T + eT^* = \bar{\Omega} + e\bar{\Omega}^*$, which was not quite as clear from Table 1.

More generally, one can interpret the column sums of the FAM as market clearing conditions (for example, T_b, R, T_h, and T_f must sum up to T), and the difference between the row sums and the column sums of the respective institutions as net worth constraints. Thus, these basic accounting constraints can be read directly off the FAM, which was not the case with balance sheets.

The position of an asset in the FAM with respect to issuer/holder does not suffice to uniquely define the asset. An important additional characteristic is the denomination of an asset (e.g. domestic or foreign currency); in addition, asset types having the same holder/issuer and denomination can have other properties (e.g. time deposits vs. demand deposits) that distinguish them.

The concept of denomination can in fact be used to distinguish between different types of assets that have the same issuer/holder combination, for example demand deposits, savings deposits, and equity that are all issued by commercial banks and held by the private sector. This is done by introducing an additional dimension into the FAM, namely "denomination". One can thus think of the FAM as a stack of matrices, each layer in the stack corresponding to a different "denomination", that is having its own price; the amount of layers being determined by the complexity of the asset structure one wishes to represent. Normally, holder, issuer, and denomination when defined solely as a choice between home and foreign currency, exhaust the types of financial assets commonly differentiated in CGE models; thus, if we denote forex-denominated assets by bold type in Table 2, the table contains a complete description of the financial asset structure we consider.

Although we express all entries in the FAM in the same units (typically the home country currency), denomination of an asset stock is important because it tells us how the value of the stock will change with revaluation, in our example meaning the change in the exchange rate. If the value of the exchange rate e increases, the value in domestic currency of the stocks typeset in bold type in Table 2 will increase. That behavior might be obvious in that particular case, as the bold entries also happen to have a factor e in front of them, but in a real-world setting all the entries would just be numbers, with no such telltales. Likewise, if we want to account for equity, we'd have to introduce a separate layer (denomination) of the FAM for each type of equity we want (e.g. home and foreign), and just like the value of "foreign currency"-denominated assets changes with the exchange rate e, the value of "domestic equity"-denominated assets would change with the price of equity. The mathematics of that turn out to be quite straightforward and are written out in Section 3.2.

A final remark on revaluation: the reader might have wondered why none of the row and column totals are in bold type, even though some of them are preceded by an e. The reason for that is that the row and column totals are not asset stocks themselves, but sums of the actual FAM entries. In our example, Taylor [2004] happened to choose the definition of Ω^* to refer to the foreign private sector's net worth *expressed in foreign currency*; but that does not imply anything about the behavior of Ω^* or $\bar{\Omega}^*$ as a result of changes in the exchange rate. The same holds for all totals. The fact that some of them, such as T^*, do happen to only consist of forex-denominated components, is a coincidence particular to our case, with no deeper meaning — for example, the Mexican government was at one point issuing both locally- and dollar-denominated bonds (technically, the latter were locally-denominated bonds indexed to the exchange rate, which amounts to the same thing for valuation purposes).

While the example we have just discussed was convenient to introduce the FAM technique, the

Holder \ Issuer	Private Sector	Commercial Banks	Central Bank	Government	Rest of the World
Private Sector		Deposits held by Private Sector, local- and **forex-**denominated	Cash held by Private Sector	Government Bonds held by Private Sector	
Commercial Banks	Loans from Banks to Private Sector, local- and **forex-**denominated		Cash and Required Deposits held by Banks	Government Bonds held by Banks	**Foreign Exchange held by Banks**
Central Bank		Rediscount		Government Bonds held by Central Bank	**Central Bank Forex Reserves**
Government	*Loans to and equity in state-owned enterprises*	*Government Deposits at Commercial Banks*	Government Deposits at the Central Bank		
Rest of the World		**Commercial Banks' Foreign Liabilities**	**Central Bank's Foreign Liabilities**	**Government's Foreign Debt**	

Table 3: *A Financial Accounting Matrix for Ghana, 1990-2002. Bold type denotes assets denominated in foreign currency*

amount of detail in each country's financial stocks was only sufficient for making a theoretical point (which was in fact the intention of Taylor [2004] when formulating the balance sheets). However, we would like to argue that the FAM methodology is just as useful when working with real-world data. Illustrating this is the purpose of the next example.

2.2 A More Realistic Example: A FAM for Ghana in the 1990s

This example is taken from the author's work on compiling a monthly time series of FAMs for Ghana, for the period 1990-2002 [Kraev 2004]. The FAM presented in Table 3 is a fairly complete representation of the Ghanaian financial sector during that period. The private sector is allowed to hold deposits, cash and government bonds, and borrows from commercial banks. Both loans and deposits can be either denominated in local currency (cedis) or in foreign currency (dollars). Commercial banks borrow cedis from the central bank, accept deposits of either denomination from the private sector, and borrow in dollars from the rest of the world. They hold the resulting assets in form of government bonds (cedi-denominated), forex reserves, cash and cedi deposits at the central banks, and loans to the private sector of either denomination. The central bank issues cash, accepts deposits from and gives loans to commercial banks as just discussed, makes loans to and accepts deposits from the government, both cedi-denominated, and has foreign assets and liabilities, both forex-denominated. As a matter of stylization, we pretend that all foreign exchange entering the country as aid or official loans is held at the central bank on the government's behalf. On the other hand, we distinguish between the central bank's foreign liabilities (roughly of the same magnitude as its forex reserves) and the government's foreign debt (a much larger and growing stock).

As in this example, just as in the last one, holder, issuer, and denomination (local/foreign) completely define the asset, we refrain from assigning symbols to assets, and instead just fill in the asset's meaning in the corresponding cell, again using bold type to denote forex-denominated stocks.

In principle, any non-diagonal cell of the FAM could be occupied (diagonal cells' values are irrelevant since they represent "institutions' liabilities towards themselves"). However in practice, in any given model only a small part of these assets will be represented, reflecting the asset structure of the

country being modeled. Thus in Table 3, private sector is not allowed to borrow from abroad or from the central bank.

It is worth emphasizing that Table 3 is not an abstract exercise. There is in fact enough data available to reconstruct monthly values of all the entries therein (except some of the government's liabilities, which are only available yearly) for the whole period 1990-2001. Thus, in addition to being a representational device, the FAM becomes a data format which we can perform analysis upon. The next two sections explain how this is done.

3 Describing Financial Accounting Matrix Dynamics

So far, the only difference of the FAM to balance sheets was how the financial stocks were arranged. We hope that we could make a good case for the simplicity, transparency, and elegance of the FAM approach. However, simplicity, transparency, and elegance are all ultimately in the eye of the beholder, and may not in themselves be sufficient reasons for adopting a new technique when one is quite comfortable with the old one.

However, once we arrange the financial stocks in a FAM format, we gain access to a matrix formalism that allows us to describe and decompose the changes in financial asset stocks over time in a fashion quite beyond any existing technique that the author is aware of.

How does the Financial Accounting Matrix evolve over time? There are three basic sources for changes in the financial stocks held by the different institution: revaluation, net lending, and capital transactions.

Revaluation refers to the change of the price of the stocks. For example, if the exchange rate changes, the value of foreign currency-denominated holdings is going to change accordingly (remember we value all financial stocks in domestic currency), with implications for net worth of both the holder and the issuer of these stocks. Likewise, if the FAM includes an "equity" denomination, then changes in the price of equity will impact the net worth of its holders and issuers.

Net lending is another kind of transaction that changes the net worth of participating agents. For example, if the government's expenses outstrip its revenue, the government must be borrowing from some other institution, generating a negative net lending flow for itself and a positive net lending flow for whoever it's borrowing from. It is the net lending flows that connect the FAM to the SAM, with the FAM replacing the "capital account" part of the SAM.

Finally, capital transactions do not change the net worth of participating agents, but rearrange the composition of their portfolios. An example is a household making a deposit at a bank: the household's deposits increase while its cash holdings decrease by the same amount, without changing its net worth; likewise the portfolio composition of the bank changes (more deposits (liabilities), more cash (assets) and, in a fractional reserve banking system, more deposits at the central bank (assets) and holdings of government bonds (assets)), but its net worth remains unaffected.

The changes to the FAM from each of these sources are described differently; after introducing some notation, we will discuss each in turn.

3.1 Notation

Let d refer to the set of available denominations. In the Ghana example, that would mean

$$d \in \{l, \$\}$$

with l meaning local currency and \$ meaning foreign currency. Let i be the set of the institutions in the FAM. In the Ghana example, that would be the *p*rivate sector, the commercial *b*anks, the *c*entral bank, the *g*overnment, and the rest of the *w*orld, abbreviated as

$$i \in \{p, b, c, g, w\}.$$

Let us denote the Financial Accounting Matrix by $\Phi_{d\,i_1 i_2}$, with the first index referring to denomination of the stock, the second to the holder (row of the matrix), and the third to the issuer (column of the matrix).

Finally, for convenience in specifying sparce matrices, let us denote by $[d : i_1 i_2]$ a 3-dimensional matrix whose all entries are zero except for the entry $(d : i_1 i_2)$, which equals one. For example, $[l : gc]$ denotes a 3-dimensional matrix such that

$$[l : gc]_{d i_1 i_2} = \begin{cases} 1 & \text{for} \quad d = l, i_1 = g, i_2 = c \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

that is, it has a one in the position corresponding to government cash holdings (or local-denominated government deposits at the central bank). Let us now use the notation in a concise description of FAM dynamics.

3.2 Revaluation

Suppose an agent holds an amount V of an asset of denomination d, so that its price in local currency is P^d. Then the value of that asset stock that appears in the FAM equals $V \cdot P^d$. If the price P^d changes, then the change in the asset stock value arising from the change in P^d is

$$\partial_t \left(VP^d\right) = \left(\partial_t P^d\right) V = \frac{\partial_t P^d}{P^d}\left(VP^d\right) = \left(\partial_t ln(P^d)\right) \cdot \left(VP^d\right) = \hat{P}^d \cdot \left(VP^d\right) \tag{2}$$

where $\hat{P}^d$ is a shorthand notation for $\partial_t ln(P^d)$, the rate of change in P^d.

Applying that to all of the FAM, we get

$$\left(\partial_t \Phi_{d\,i_1 i_2}\right)_{from\ revaluation} = \hat{P}^d \Phi_{d i_1 i_2} \tag{3}$$

In the example of a FAM for Ghana, where there are only two denominations, local currency l and foreign exchange \$, so

$$\hat{P}^d = \begin{cases} 0 & \text{for} \quad d = l \\ \hat{e} & \text{for} \quad d = \$ \end{cases} \tag{4}$$

where $\hat{e}$ is the depreciation rate of the local currency.

On the other hand, if we were putting together a FAM for the US, the denomination index set could be, for example,

$$d \in \{\$, \varepsilon, €\},$$

thus encompassing dollar-denominated assets, domestic equity (having dollar price index $P^\varepsilon = qPK/E$ in the notation of Table 1, with E denoting the number of outstanding equity shares); and euro-denominated assets whose dollar price index is the exchange rate $P^{€} = e^{€/\$}$. Instead of (4) we would then have

$$\hat{P}^d = \begin{cases} 0 & \text{for} \quad d = \$ \\ \hat{q} + \hat{P} + \hat{K} - \hat{E} & \text{for} \quad d = \varepsilon \\ \widehat{e^{€/\$}} & \text{for} \quad d = € \end{cases} \tag{5}$$

3.3 Current Transactions and Net Lending

The actual current transactions – purchases of products, transfers, taxes, etc. – are carried out in the SAM. What goes into the FAM is the net resulting change in each agent's *financial* net worth, that is, each agent's net lending (saving minus investment for the private sector, budget surplus for the government, etc.), which we will denote by S_i. As S_i is the only "leakage" from an otherwise leak-less SAM, Walras' Law implies

$$\sum_i S_i = 0 \tag{6}$$

that is, the sum of net lending over all institutions is identically zero.

Note that whenever we say "net worth" in this paper, we mean "financial net worth", that is we do not count stocks of physical capital towards net worth, and thus all purchases, even of capital goods such as machinery or buildings, or of real estate, count as "current transactions". This is convenient to do because these are also handled in the SAM, so that here we can concentrate exclusively upon financial stock dynamics.

The question now is how to feed these net lending flows into the FAM, as each agent has different asset stocks that these net lending flows could potentially feed into. The way we choose to tackle this is to have the net lending flows feed into each institution's cash balances, i.e. its stock of central bank-issued, local currency-denominated liabilities. All institutions are then assumed to re-balance their portfolios according to whatever portfolio preferences they have. Thus the change to the FAM from current transactions can be written as

$$\partial_t \Phi_{from\ current\ transactions} = \sum_i S_i \cdot [l : ic]. \tag{7}$$

Feeding the net lending flows into the cash holdings in this way is justified for two reasons: firstly, a large part of the transactions are indeed cash-based; and secondly, those institutions that enter non-cash transactions are likely to rebalance their portfolio between different assets much faster than the current transactions can happen (financial markets clear a lot faster than real goods do), so it doesn't really matter which of their assets their net lending is hooked up to.

The final question with this scheme is whether it also works for the central bank, as its liabilities towards itself do not change its net worth. But as we mentioned above, $\sum_i S_i = 0$ and thus the change in net worth of the central bank from the other terms in (7) (that change the amount of central bank's liabilities held by other institutions) will exactly equal S_c.

Thus the system is consistent, and we are ready to discuss the final and most interesting source of changes in the FAM, namely capital transactions, such as portfolio rebalancing.

3.4 Capital Transactions and Transaction Matrices

The final source of changes in the FAM are capital transactions. These have two properties that can complicate models: firstly, any capital transaction affects several entries in the FAM; secondly, these always change in such a manner that the net worth of the institutions involved is unchanged. The first of these properties is liable to lead to a proliferation of equations, the second is a source of constraints that have to be watched, lest errors arise in these equations.

Let us illustrate that in a simple example: a firm taking up a local-currency denominated loan from a commercial bank. For each dollar of the loan value, the amount of the firm's debt stock $\Phi_{l\,bf}$ increases by a dollar, and so does the amount of the firm's deposits at that bank $\Phi_{l\,fb}$ (we assume the loan takes the form of deposits, rather than a cash payout to the firm). This is a very simple example; in the case of deposit creation, households give cash to banks and acquire deposits there, but then the bank deposits some of that cash at the central bank (primary deposit requirement) and buys government bonds for a further share of the cash (secondary deposit requirement), so that even more FAM stocks are affected as a result of one rather simple transaction.

How can we describe these changes in a simple way, with preferably only one equation per transaction? We propose to do so by introducing a new formalism that we call transaction matrices. It is based on the observation that while a transaction, such as creation of new loans, affects several entries in the FAM, the impact of the transaction on all of them is *proportional to the amount of the transaction.*

To further elaborate on our example of loan creation, let us define a matrix Λ^{loan}, as pictured in Table 4.

Using the notation we introduced earlier, this matrix can also be written as

$$\Lambda^{loan} = [l : bf] + [l : fb] \tag{8}$$

$\Lambda^{loan}_{li_1i_2}$	h	f	b	c	g
h					
f			1		
b		1			
c					
g					

Table 4: *The transaction matrix for creation of new local-currency denominated loans (the local-denominated component; the forex-denominated component is identically zero)*

Suppose the banks make a new loan of size ψ_{loan}. Then from our brief discussion of the process of loan creation above we see that the total change in Φ due to that loan equals

$$(\Delta\Phi_{d\,i_1i_2})_{from\ loans\ to\ firms\ in\ local\ currency} = \Lambda^{loan}_{d\,i_1i_2} \cdot \psi_{loan}$$

We thus see that with the help of the transaction matrix, we only need one (matrix) equation to represent one transaction, regardless of the number of stocks affected.

In addition to that, properly constructed transaction matrices automatically make sure that capital transactions do not affect the net worth of the institutions involved. "Properly constructed" here means simply that each transaction matrix Λ must fulfill

$$\sum_{d,i_1} \Lambda_{d\,i_1i} - \sum_{d,i_1} \Lambda_{d\,ii_1} = 0 \quad \text{for all } i \tag{9}$$

Thus the transaction matrix formalism to describe financial stocks solves both problems outlined above, allowing for a simple representation of financial stocks in a dynamic model. If we have a transaction matrix for each transaction allowed in the model, and use the index λ to number them and the number ψ_λ to describe the amount of a transaction of type λ happening at a given moment, then the changes in the FAM from capital transactions can be written simply as

$$(\partial_t\Phi_{d\,i_1i_2})_{from\ capital\ transactions} = \sum_\lambda \psi_\lambda \Lambda^\lambda_{d\,i_1i_2} \tag{10}$$

This is an extremely useful decomposition as it effectively separates the *structure* of the financial sector from the *behavior* of the institutions therein. The transaction matrices $\Lambda^\lambda_{d\,i_1i_2}$ are constant and describe the kinds of transactions that are *possible* in a given economy ("Can households hold foreign exchange? Can firms borrow abroad?") whereas the "transaction flows" ψ_λ describe the decisions of individual agents as to which of the possible transactions they actually *want* to undertake. Hereby the net worth constraint of every institution is automatically observed.

The only thing left to watch for is that none of the entries of Φ should be allowed to become negative - essentially a boundary condition; all the net wealth constraints are observed automatically given that all transaction matrices are "properly constructed", that is,

$$\sum_{d,i_1} \Lambda^\lambda_{d\,i_1i} - \sum_{d,i_1} \Lambda^\lambda_{d\,ii_1} = 0 \quad \text{for all } \lambda, i. \tag{11}$$

What do transaction matrices look like? The next subsection presents a complete set of transaction matrices for our Ghana example.

3.5 A Realistic Set of Transaction Matrices for Ghana in the 1990s

Table 5 shows a complete set of Transaction Matrices for the FAM of Table 3. The first column gives the TMs a one-letter name for later use, the second column explains the meaning of the particular TM, and the third defines the TM in the notation we have introduced.

A	Private Creation of Local Currency-Denominated Deposits	$[l:pb]-[l:pc]+[l:bg]-[l:cg]$
B	Private Purchases of Government Bonds	$-[l:pc]+[l:pg]-[l:cg]$
C	Private Creation of Forex-Denominated Deposits	$-[l:pc]+[l:bg]-[l:cg]+[\$:pb]$
D	Private Local Currency-Denominated Borrowing from Comm. Banks	$[l:pc]+[l:bp]-[l:bg]+[l:cg]$
E	Private Forex-Denominated Borrowing from Commercial Banks	$[l:pc]-[l:bg]+[l:cg]+[\$:bp]$
F	Commercial Banks' Purchases of Government Bonds	$-[l:bc]+[l:bg]-[l:cg]$
G	Commercial Banks' Borrowing from Abroad	$[\$:bw]+[\$:wb]$
H	Central Bank Exchanging Currency	$-[l:wc]-[\$:cw]$
I	Comm. Banks Borrowing in Local Currency from the Central Bank	$[l:cb]+[l:bc]$
J	Commercial Banks' Purchases of Forex from the Central Bank	$-[l:bc]+[\$:bw]-[\$:cw]$
K	Central Lending to Government	$[l:cg]+[l:gc]$
L	Central Bank Borrowing From Abroad	$[\$:cw]+[\$:wc]$
M	Government Foreign Borrowing	$[l:gc]+[\$:cw]+[\$:wg]$
N	Renormalization of Government Deposits at Commercial Banks	$[l:bc]+[l:gb]-[l:gc]$
O	Central Bank Lending to Government-Owned Enterprises	$[l:pc]+[l:cp]$

Table 5: *Transaction Matrices for the Ghana FAM*

Let us talk our way through one of these to see how they are constructed. Take the first one, "Private Creation of Local Currency-Denominated Deposits". What happens when a unit of cash is deposited at a commercial bank? The private sector has one less unit of cash ($-[l:pc]$) but one more unit of local-denominated deposits ($[l:pb]$). The bank that gets the unit of cash uses it to buy a unit of government bonds ($[l:bg]$) from the central bank ($-[l:cg]$), because bonds attract interest rates and cash doesn't. After that, the bank may re-balance its portfolio using other TMs under its control, but what we have just described is the unit transaction of deposit creation. The other TMs are composed in a similar manner.

While the TM collection as a whole must be "just large enough" to explain FAM dynamics in a given economy, there seems to be a substantial degree of freedom in choosing individual TMs. For example, while defining TM A, we could have let the bank keep the cash as cash instead of converting it into bonds, arriving at a matrix $[l:pb]-[l:pc]+[l:bc]$. It is not at this point clear to the author whether there is a good reason to prefer one of these formulations to the other, and readers' suggestions would be welcome.

3.6 The Master Equation for FAM Dynamics

Now that we have understood how to model each of the sources of changes in the FAM in Sections 3.2-3.4, we can pull them together to formulate the master equation describing the FAM dynamics:

$$\partial_t \Phi_{di_1i_2} = \hat{P}^d \Phi_{d\,i_1i_2} + \sum_i S_i \cdot [l:ic] + \sum_\lambda \psi_\lambda \Lambda^\lambda_{d\,i_1i_2} \tag{12}$$

Here $\hat{P}^d$ is the rate of change of the local currency price of stock denomination d, S_i are the net lending flows coming from the SAM, ψ_λ are the amounts of transactions actually undertaken by the institutions, and $\Lambda^\lambda_{d\,i_1i_2}$ are the transaction matrices describing how each transaction affects the different flows in the FAM.

This is sufficient for a constructive description of FAM dynamics in a SAM/FAM model. The asset price behavior, determined elsewhere in the model, will determine the first term, the net lending vectors will come from the SAM, and the transaction flows ψ_λ can be whatever the agents who control

them want them to be - the transaction matrix formalism will automatically enforce all the net worth constraints that would have to be imposed as additional equations had we wanted to directly specify the dynamics of the individual FAM entries.

The next section shows tackles the inverse problem, namely the efficient decomposition of time series of FAM data into the form (12).

4 Decomposing Observed FAM Time Series

We have just seen how, given an initial value of the FAM together with time series for the exchange rate and the net lending of each institution, as well as the values of the transaction flows, we can reconstruct the time path that the FAM follows.

Sometimes, however, we may be interested in the opposite operation: given the values of the FAM for every moment in time (that we have assembled from bank and government statistics) and asset price time series, decompose the FAM changes into revaluation, net lending, and capital transactions. Once we have achieved that, we could seek to describe the observed transaction flows in terms of portfolio-equilibrating behavior of the institutions, stylize the FAM while preserving everybody's net worth by omitting or modifying some of the transaction flows, and play many other interesting games.

4.1 The General Case

The revaluation term is directly determined from asset price series and FAM values in Section 3.2. The question then becomes: given a set of transaction matrices $\Lambda^{\lambda}_{d\,i_1 i_2}$, time series for the FAM $\Phi_{d i_1 i_2}(t)$ and the asset prices expressed in local currency $P^d(t)$, find $S_i(t)$ and $\psi_\lambda(t)$.

The first part, namely finding $S_i(t)$, is simple. Let us define the revaluation-corrected change in Φ as

$$D\Phi_{di_1 i_2} = \partial_t \Phi_{di_1 i_2} - \hat{P}^d \Phi_{di_1 i_2} \tag{13}$$

Then from (12) we follow

$$D\Phi_{di_1 i_2} = \sum_i S_i \cdot [l : ic] + \sum_\lambda \psi_\lambda \Lambda^{\lambda}_{di_1 i_2} \tag{14}$$

and since all the transaction matrices are properly constructed, the first term in (14) is the only one that contributes to changes in institutions' net worth. Thus we can find S_i as the revaluation-corrected change in net worth,

$$S_i = \sum_{d,i_1} (D\Phi_{dii_1} - D\Phi_{di_1 i}) \tag{15}$$

The second part of the problem, namely determining ψ_λ, turns out to require an additional trick. Representing "any" net worth-preserving (after we have cleaned away the other terms) change in the FAM through a linear combination of transaction matrices is clearly only possible if we have "enough" transaction matrices. This representation will be unique if we have "just enough" transaction matrices rather than "too many". Roughly, this means that we need as many TMs as there are nonzero non-cash entries in the FAM. To make that statement more precise, we will need some linear algebra.

Let V be the space of all "valid" FAMs, i.e. all FAMs that could in principle occur in a given

economy. For $X^1, X^2 \in V$, let us define a "pseudoproduct"[1]

$$\left\langle X^1, X^2 \right\rangle = \sum_{d,i_1,i_2} X^1_{d,i_1,i_2} X^2_{d,i_1,i_2} - \sum_i X^1_{l,i,c} X^2_{l,i,c} \tag{16}$$

This means that $\langle X^1, X^2 \rangle$ is the sum of pairwise products of all entries of the two matrices except the local-denominated cash entries. If we had left out the second sum (the one after the minus sign) in (16), we would have had the canonical scalar product on the space of all FAMs - that is, the exact same expression that we would have gotten by writing out all elements of each FAM as a very long vector (with as many elements as there are entries in a FAM) and computing the "regular" scalar product of the two vectors.

With the inclusion of the second sum, (16) is equivalent to first dropping the local-denominated cash terms (local-denominated central bank liabilities) and *then* flattening out the rest of both FAMs into vectors and computing the "regular" scalar product.

Let us compute the pseudoproduct of both sides of (14) with an arbitrary transaction matrix Λ^{λ_1}:

$$\begin{aligned}
\left\langle \Lambda^{\lambda_1}, D\Phi \right\rangle &= \left\langle \Lambda^{\lambda_1}, \sum_i S_i \cdot [l : ic] \right\rangle + \left\langle \Lambda^{\lambda_1}, \sum_\lambda \psi_\lambda \Lambda^\lambda \right\rangle && (17)\\
&= \sum_i S_i \cdot \left\langle \Lambda^{\lambda_1}, [l : ic] \right\rangle + \sum_\lambda \psi_\lambda \left\langle \Lambda^{\lambda_1}, \Lambda^\lambda \right\rangle && (18)\\
&= \sum_\lambda \psi_\lambda \left\langle \Lambda^{\lambda_1}, \Lambda^\lambda \right\rangle && (19)
\end{aligned}$$

Here the second equality holds because the pseudoproduct ignores local-denominated liabilities of the central bank, and $[l : ic]$ has no other terms; thus the pseudoproduct of $[l : ic]$ with *anything* is identically zero (in fact, the whole point of introducing the pseudoproduct was for this to be the case).

We are now almost finished. Let the matrix Θ contain the pairwise pseudoproducts of the transaction matrices:

$$\Theta^{\lambda_1 \lambda} = \left\langle \Lambda^{\lambda_1}, \Lambda^\lambda \right\rangle, \tag{20}$$

and let the vector c contain the pseudoproducts of each of the transaction matrices with $D\Phi$

$$c^{\lambda_1} = \left\langle \Lambda^{\lambda_1}, D\Phi \right\rangle, \tag{21}$$

then (19) can be written in matrix notation as

$$c = \Theta \psi \tag{22}$$

which (if Θ is not singular) is easily solved as

$$\psi = \Theta^{-1} c \tag{23}$$

[1]We call it a pseudoproduct because it is almost, but not quite, a scalar product. In fact, the whole decomposition below hinges on this particular choice of pseudoproduct. Why do we define it that way?

Most readers will probably be satisfied with knowing that wherever the definition comes from, it works to produce the decomposition (24) below. However, for the more mathematically inclined of our readers, here is a glimpse of the theoretical underpinnings:

What we want is to take away the effects of the terms $[l : ic]$ from (14). Let W denote the space of all linear combination of these terms, that is the space of all FAMs whose only nonzero terms are local-denominated central bank liabilities. Then (16) is the natural projection of the canonical scalar product on V onto the quotient space V/W; the precise meaning of having "exactly enough" TMs is that the TMs have to form a basis of V/W; and (24) is simply a change of basis in V/W.

For an introduction to the relevant linear algebra concepts, we recommend the delightful textbook by Michael Artin [1991].

or, if we write it out for each component,

$$\psi_\lambda = \sum_{\lambda_1} \Theta^{-1}_{\lambda\lambda_1} c^{\lambda_1} = \sum_{\lambda_1} \Theta^{-1}_{\lambda\lambda_1} \left\langle \Lambda^{\lambda_1}, D\Phi \right\rangle . \tag{24}$$

This decomposition is computationally efficient because Θ^{-1} is a constant matrix (it is obtained from the constant transaction matrices). Thus the only computation we need to do at each time step is to compute the pseudoproduct of $D\Phi$ with each TM, and then multiply the resulting vector by the constant matrix Θ^{-1}.

Thus the transaction matrix decomposition is made constructive. This construction also gives us a practical answer to the question "is a given set of TM's enough? Exactly enough?". The answer is as follows: if the matrix Θ in (20) is singular and thus won't invert, the transaction matrices are not linearly independent, and one should omit some. If Θ inverts, one defines ψ_λ by (23) and then computes the residual w from (14) as

$$w = D\Phi_{di_1i_2} - \sum_i S_i \cdot [l : ic] - \sum_\lambda \psi_\lambda \Lambda^\lambda_{di_1i_2} \tag{25}$$

If w is zero (to numerical precision), we have enough transaction matrices; if not, we need to add some more to explain the other entries in the residual.

A question worth discussing here is: what reason do we have to assume that the matrix Θ is not singular? First, note that it is reasonable to suppose that we have "enough" transaction matrices (TMs) so that a decomposition of the form (14) is always possible. After all, we construct the TMs, so that if we don't have enough to explain any given FAM behavior, we can always introduce some more. In that case, non-singularity of Θ is equivalent to saying that the decomposition (14) is *unique*.

The existence of a unique decomposition (14) is in turn equivalent to saying the transaction matrices, together with the matrices $[l : ic]$, form a basis of the space V of all possible FAMs for the economy in question. Is that a reasonable thing to expect? That is, does it *make sense* to have just enough TMs for a *unique* decomposition ? While it definitely seems wise to use *enough* transaction matrices to enable us to span the changes in the financial stock composition allowed by the structure of the economy, it is not as clear whether one should use "just enough" of them, or instead use a larger, linearly dependent set.

Since in the latter case one could select a basis out of the larger set and represent the rest of the TMs as linear combinations thereof, the question seems to be largely one of style. If one is building a toy model and for some reason wants to allow lots of redundant transactions, there does not seem to be an a priori reason not to. However, if one wants to start from data and have a unique decomposition, as we do, it seems wise to keep just enough TMs, as we do in our example.

4.2 Transaction Matrix Decomposition in Our Example

The matrices Θ and Θ^{-1} generated from the transaction matrices of Table 5 are shown in Tables 6 and 7, respectively.

Θ and Θ^{-1} are square matrices, with rows and columns of each matrix each going over the transaction matrices specified in Table 5, named A to O. It so happens that the determinant of Θ equals 1, so all entries of Θ^{-1} are integers. However, there is no reason to expect this to happen in the general case.

To illustrate how the pseudoproduct works in practice, let us compute a couple of the entries in Table 6 by hand to show how it's done. From (16) we obtain the following algorithm for computing pseudoproducts: let us take two transaction matrices, say A and C, in the representation of Table 5. First, we discard all terms that are local-denominated liabilities of the central bank (that is, look like $[l : _c]$), which in this case means $[l : pc]$ for both TMs. Then we discard all terms except those that appear in *both* TMs (whatever their coefficients are), leaving us in this case with the terms containing $[l : bg]$ and $[l : cg]$ for both matrices. Then, just as with a regular scalar product, we compute the

Θ	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
A	3	1	2	-2	-2	2					-1				
B	1	2	1	-1	-1	1					-1				
C	2	1	3	-2	-2	2					-1				
D	-2	-1	-2	3	2	-2					1				
E	-2	-1	-2	2	3	-2					1				
F	2	1	2	-2	-2	2					-1				
G							2			1					
H								1		1		-1	-1		
I									1						
J							1	1		2		-1	-1		
K	-1	-1	-1	1	1	-1					1				
L								-1		-1		2	1		
M								-1		-1		1	2		
N														1	
O															1

Table 6: *Pairwise pseudoproducts of transaction matrices*

Θ^{-1}	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
A	1					-1									
B		1									1				
C			1			-1									
D				1		1									
E					1	1									
F	-1		-1	1	1	5					1				
G							1	1		-1					
H							1	5		-2		1	1		
I									1						
J							-1	-2		2					
K		1				1					3				
L								1				1			
M								1					1		
N														1	
O															1

Table 7: *The inverse of the pairwise TM pseudoproduct matrix* Θ

pairwise products of the corresponding terms' coefficients and add all these products up, meaning in our case $(-1 \times -1) + (1 \times 1) = 2$.

As all coefficients in Table 5 are either 1 or -1, we can see from the above algorithm that the pseudoproduct of a TM with itself equals the number of terms in that TM's definition, not counting local-currency central bank liabilities. This is again easily checked by comparing Table 5 with the diagonal of Table 6.

The above matrices were in fact used to decompose the monthly FAM series for Ghana for 1990-2002 using equations (13), (15), and (24), and then re-compose it using equation (12). At each time step, we computed the difference between the original $D\Phi$ and the reconstructed $D\Phi$. The sum of the squares of all the entries of the difference was then divided by the sum of squares of all the entries of Φ, to define the relative square error for that time step. The meaning of the relative square error is how large the reconstruction error is compared to the actual entries of the FAM being reconstructed. This relative square error was then cumulated for all 156 time steps of the simulation, giving the end value of $2.1 \cdot 10^{-14}$, which is zero to numerical precision. This confirms that the analysis and reconstruction formulas described here are indeed exact.

5 Discussion

The Financial Accounting Matrix/Transaction Matrix formalism that we have introduced allows us to decompose the changes in the financial stocks into revaluation, current transactions, and capital transactions in a computationally efficient way. Conversely, the transaction matrix formalism allows us to directly specify the portfolio allocation behavior of the different institutions while *automatically* observing the wealth constraints of the institutions.

The FAM/TM formalism greatly simplifies the specification and estimation of portfolio behavior in stock-flow consistent models, as well as allowing elegant integration of a Social Accounting Matrix describing the nominal flows in the economy with a Financial Accounting Matrix describing the gross asset stocks.

Besides being interesting in its own right, the decomposition of FAM time series according to the master equation (12) allows us to get more reliable estimates for net financial savings (net lending) of each institution in a developing country context. In an economy where some important sectors are demand-driven (that is, probably, any economy), knowledge of net lending is important to account for injections and leakages contributing to the aggregate demand. Unfortunately, net lending in a SAM is often computed as a residual from flow data, and is thus quite unreliable; we would argue that using asset stocks for that purpose makes for a much more reliable estimate.

The reason for that is that in developing countries, financial asset stock time series are available and generally more reliable than most flow data. This is firstly, because stock data are more easily observed than flow data (as a point observation is enough to pin down a stock variable, while we have to observe a flow variable over a period of time to measure it); secondly, because unlike data on physical quantities, most financial stocks are automatically measured, the numbers in a bank's computer *defining* the corresponding stock; and finally, because typically the banking sector is the most modernized part of an economy in terms of data collection and processing. This is true for both the private banking sector and the central bank. Therefore, in addition to its power in describing financial stock dynamics, the FAM/TM formalism allows for a more reliable SAM estimation for developing countries.

The situation is in a sense reversed in the developing countries such as the United States. There, flow of funds data give us net lending flows as well as nominal values of stocks at the beginning and end of period, allowing us to deduce revaluation and thus asset price movements.

Thus, in both industrialized and developing countries, there is sufficient data to operationalize the decomposition (12). Once that is done, the consistent FAM/SAM time series will provide a transparent, complete data base for structural analysis and model estimation in a given economy.

Another feature of the Transaction Matrix decomposition worth remarking on is that it is easily adaptable to a changing financial sector structure. For ease of exposition, we assumed the Transaction Matrices to be constant throughout the paper, and this will indeed be sufficient for most purposes. However, in the case when new transactions are allowed in the middle of the period under study, say because of financial sector liberalization, we can just add new transaction matrices to the set. This means that the matrix Θ will become time-dependent; but as the decomposition (24) happens separately for every time step, all the formulae derived here will still work.

Summing up, the Financial Accounting Matrix/Transaction Matrices is a simple yet powerful way to describe financial stock dynamics. It cleanly separates universal accounting constraints, country-specific financial sector structure, and behavior of agents within that structure, paving the way for data analysis and modeling that is at the same time more elegant and more realistic than is feasible with existing notation.

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