Chapter 8

Decomposing the FAM dynamics

How does the Financial Accounting Matrix Φ evolve over time? There are three basic sources for changes in the financial stocks held by the agents: revaluation, current transactions, and capital transactions. Revaluation refers to the change of the price of the stocks. In our case, if the exchange rate changes, the value of foreign currency-denominated holdings is going to change accordingly (remember we value all financial stocks in domestic currency). Current transactions refer to transactions that change the net worth of participating agents. For example, if the government pays wages to its employees, the net worth of the government decreases and that of the private sector increases. Finally, capital transactions do not change the net worth of participating agents. An example is a household making a deposit at a bank: the household's deposits increase while its cash holdings decrease by the same amount, without changing its net worth; likewise the asset composition of the bank changes (more deposits (liabilities), more cash (assets) and probably more deposits at the central bank (assets)), but its net worth remains unaffected. The changes to the FAM from each of these sources are described differently; let us discuss each in turn.

8.1 Revaluation

Suppose an agent holds an amount V of an asset of denomination d, so that its price in local currency is P^d. Then the value of that asset stock that appears in the SAM equals $V \cdot P^d$. If the price P^d changes, then the change in the asset stock value arising from the change in P^d is

$$\partial_t \left(VP^d\right) = \left(\partial_t P^d\right) V = \frac{\partial_t P^d}{P^d}\left(VP^d\right) = \left(\partial_t ln(P^d)\right) \cdot \left(VP^d\right) = \hat{P}^d \cdot \left(VP^d\right) \tag{8.1}$$

where $\hat{P}^d$ is a shorthand notation for $\partial_t ln(P^d)$.

Applying that to all of the FAM, we get

$$(\partial_t \Phi_{di_1i_2})_{from\ revaluation} = \hat{P}^d \Phi_{di_1i_2} \tag{8.2}$$

In our case there are only two denominations, local currency l and foreign exchange \$, so

$$\hat{P}^d = \begin{cases} 0 & \text{for} \quad d = l \\ \hat{e} & \text{for} \quad d = \$ \end{cases} \tag{8.3}$$

where $\hat{e}$ is the depreciation rate of the local currency.

8.2 Current Transactions

The actual current transactions – purchases of products, transfers, taxes, etc. – are carried out in the SAM. What goes into the FAM is the *net* resulting change in each agent's net worth, that is, each agent's lending, which we have earlier denoted by S_i. Note that whenever we say "net worth" in this chapter, we mean "financial net worth", that is we do not count stocks of physical capital towards net worth, and thus all purchases, even of capital goods such as machinery or buildings, count as "current transactions". This is convenient to do because these are also handled in the SAM, so that here we can concentrate exclusively upon financial stock dynamics.

The question now is how to feed them into the FAM, as each agent has multiple different asset stocks that these net lending flows could potentially feed into. The way we choose to tackle this is to have the net lending flows feed into each institution's cash balances, i.e. its stock of central bank-issued local currency-denominated liabilities. All institutions are then assumed to re-balance their portfolios according to whatever portfolio preferences they have. Thus the change to the FAM from current transactions can be written as

$$\partial_t \Phi_{from\ current\ transactions} = \sum_i S_i \cdot [lic]. \tag{8.4}$$

Here $[lic]$ is used to denote a 3-dimensional matrix such that

$$[lic]_{di_1i_2} = \begin{cases} 1 & \text{for} \quad d = l, i_1 = i, i_2 = c \\ 0 & \text{otherwise} \end{cases} \tag{8.5}$$

Feeding the net lending flows into the cash holdings in this way is justified for two reasons: firstly, a large part of the transactions are indeed cash-based; and secondly, those institutions that enter non-cash transactions are likely to rebalance their portfolio between different assets much faster than the current transactions can happen (financial markets clear a lot faster than real goods do), so it doesn't really matter which of their assets their net savings are hooked up to.

The final question with this scheme is whether it also works for the central bank, as its liabilities towards itself do not change its net worth. But as we mentioned in Chapter 5, since S_i is the only leakage from an otherwise leak-less SAM, we always have

$$\sum_i S_i = 0 \tag{8.6}$$

and thus the change in net worth of the central bank from the other net lending flows (that change the amount of central bank's liabilities held by other institutions) will exactly equal S_c. Thus the system is consistent, and we are ready to discuss the final and most interesting source of changes in the FAM, namely capital transactions, such as portfolio rebalancing.

8.3 Capital Transactions

The final source of changes in the FAM are capital transactions. These have two properties that can complicate models: firstly, any capital transaction affects several entries in the FAM; secondly, these always change in such a manner that the net worth of the institutions involved is unchanged. The first of these properties is liable to lead to a proliferation of equations, the second is a constraint that has to be watched, unless errors arise in these equations.

Let us illustrate that in a simple example: a firm taking up a local-currency denominated loan from a local commercial bank. For each dollar of the loan value, the amount of the firm's debt stock (Φ_{lbf}) increases by a dollar, and so does the amount of the firm's deposits at that bank (we assume the loan takes the form of deposits, rather than a cash payout to the firm). This is a very simple example; in the case of deposit creation, households give cash to banks and acquire deposits there, but then the bank deposits some of that cash at the central bank (primary deposit requirement) and buys government bonds for a further share of the cash (secondary deposit requirement), so that multiple FAM stocks are affected as a result of one rather simple transaction.

How can we describe these changes in a simple way, with preferably only one equation per transaction? We propose to do so by introducing a new formalism that we call transaction matrices. It is based on the observation that while a transaction, such as creation of new loans, affects several entries in the FAM, the impact on all of them is proportional to the amount of the transaction. To further elaborate on our example of loan creation, let us define a matrix Λ^{loan}, as pictured in Table 8.1. Using the notation we introduced earlier, this matrix can also be written as

$\Lambda^{loan}_{li_1i_2}$	h	f	b	c	g
h					
f			1		
b		1			
c					
g					

Table 8.1: *The transaction matrix for creation of new local-currency denominated loans*

$$\Lambda^{loan} = [lbf] + [lfb] \tag{8.7}$$

Suppose the banks make a new loan of size ψ_{loan}. Then from our brief discussion of the process of loan creation above we see that the total change in Φ due to that loan equals

$$(\Delta\Phi_{li_1i_2})_{from\ loans\ to\ firms\ in\ local\ currency} = \Lambda^{loan}_{li_1i_2} \cdot \psi_{loan}$$

We thus see that with the help of the transaction matrix, we only need one (matrix) equation to represent one transaction. In addition to that, properly constructed transaction matrices will automatically make sure that capital transactions do not affect the net worth of the institutions involved. "Properly constructed" here means simply that each transaction matrix Λ must fulfill

$$\sum_{i_1} \Lambda_{i_1 i} - \sum_{i_1} \Lambda_{i i_1} = 0 \quad \text{for all } i \tag{8.8}$$

Thus the transaction matrix formalism to describe financial stocks solves both problems outlined above, allowing for a simple representation of financial stocks in a dynamic model. If we have a transaction matrix for each transaction allowed in the model, and use the index λ to number them and the number ψ_λ to describe the amount of a transaction of type λ happening at a given moment, then the changes in the FAM from capital transactions can be written simply as

$$(\partial_t \Phi_{di_1i_2})_{from\ capital\ transactions} = \sum_\lambda \psi_\lambda \Lambda^\lambda_{di_1i_2} \tag{8.9}$$

This is an extremely useful decomposition as it effectively separates the structure of the financial sector from the behavior of the institutions therein. The transaction matrices $\Lambda^{\lambda}_{di_1i_2}$ are constant and describe the kinds of transactions that are *possible* in a given economy ("Can households hold foreign exchange? Can firms borrow abroad?") whereas the "transaction flows" ψ_λ describe the decisions of individual agents as to which of the possible transactions they actually want to undertake. Hereby the net worth constraint of every institution is automatically observed.

The only thing left to watch for is that none of the entries of Φ should be allowed to become negative - essentially a boundary condition; but all the "accounting constraints" are observed automatically given that all transaction matrices are "properly constructed".

8.4 The Master Equation for FAM Dynamics

Now that we have understood how to model each of the sources of changes in the FAM, we can pull them together to formulate the master equation describing the FAM dynamics:

$$\partial_t \Phi_{di_1i_2} = \hat{P}^d \Phi_{di_1i_2} + \sum_i S_i \cdot [lic] + \sum_\lambda \psi_\lambda \Lambda^{\lambda}_{di_1i_2} \tag{8.10}$$

This is sufficient for a constructive description of FAM dynamics in a SAM/FAM model. The exchange rate behavior, determined elsewhere in the model, will determine the first term, the net lending vectors will come from the SAM, and the transaction flows ψ_λ can be whatever the agents who control them want them to be - the transaction matrix formalism will automatically enforce all the net worth constraints that would have to be imposed as additional equations had we wanted to directly specify the dynamics of the individual FAM entries.

8.5 Reconstructing Transaction Flows From FAM Time Series

We have just seen how, given an initial value of the FAM together with time series for the exchange rate and the net lending of each institution, as well as the values of the transaction flows, we can reconstruct the time path that the FAM follows. As our approach starts with data, however, we are also interested in the opposite operation: given the values of the FAM for every moment in time (that we have assembled from Central Bank statistics) and exchange rate time series, decompose the FAM changes into revaluation, net lending, and capital transactions. Once we have achieved that, we can seek to describe the transaction flows in terms of portfolio-equilibrating behavior of the institutions, stylize the FAM while preserving everybody's net worth by omitting or modifying some of the flows, and play many other interesting games.

The question thus is: given a set of transaction matrices $\Lambda^{\lambda}_{di_1i_2}$ and time series for the FAM $\Phi_{di_1i_2}(t)$ and the exchange rate $e(t)$, find $S_i(t)$ and $\psi_\lambda(t)$.

The first part, namely finding $S_i(t)$, is simple. Let us define the revaluation-corrected change in Φ as

$$D\Phi_{di_1i_2} = \partial_t \Phi_{di_1i_2} - \hat{P}^d \Phi_{di_1i_2} \tag{8.11}$$

Then

$$D\Phi_{di_1i_2} = \sum_i S_i \cdot [lic] + \sum_\lambda \psi_\lambda \Lambda^{\lambda}_{di_1i_2} \tag{8.12}$$

and since all the transaction matrices are properly constructed, the first term is the only one that contributes to changes in institutions' net worth. Thus we can find S_i as the change in revaluation-corrected net worth,

$$S_i = \sum_{d,i_1} (D\Phi_{dii_1} - D\Phi_{di_1i}) \tag{8.13}$$

This is exactly how we determined S_i in Chapter 7, Equation (7.31).

The second part of the problem, namely determining ψ_λ, turns out to require a bit more theory, but is quite cheap in computational terms. First of all, representing "any" net worth-preserving (after we have cleaned away the other terms) change in the FAM through a linear combination of transaction matrices is clearly only possible if we have "enough" transaction matrices. This representation will be unique if we have "just enough" transaction matrices rather than "too many". Roughly, this means that we need as many TMs as there are nonzero non-cash entries in the FAM. To make that statement more precise, we will need some linear algebra.

First of all, note that the decomposition (8.12) is unique iff the TMs are linearly independent. We can re-cast the problem (8.12) as "decompose $D\Phi$ into a linear combination of TMs up to an error in the local cash terms". Now let V be the space of all "valid" FAMs, i.e. all FAMs that could in principle occur in a given economy. Let W be the five-dimensional space of all combinations of local cash terms only, i.e.

$$W = span\{[lic] | i \in Inst\} \tag{8.14}$$

Then let us define a semi-positive definite scalar product on V that is zero exactly over W. Let $X^1, X^2 \in V$, then

$$\left\langle X^1, X^2 \right\rangle = \sum_{d,i_1,i_2} X^1_{d,i_1,i_2} X^2_{d,i_1,i_2} - \sum_i X^1_{l,i,c} X^2_{l,i,c} \tag{8.15}$$

This means that $\langle X^1, X^2 \rangle$ is the sum of pairwise products of all entries of the two matrices except the local-denominated cash entries. If we had left out the second sum (the one after the minus sign) in (8.15), we would have had the canonical scalar product on the space of all FAMs - that is, the exact same expression that we would have gotten by writing out all elements of each FAM as a very long vector (with as many elements as there are entries in a FAM) and computing the "normal" scalar product of the two vectors.

With the inclusion of the second sum, (8.15) is equivalent to dropping the local-denominated cash terms (local-denominated central bank liabilities) and *then* flattening out both FAMs into vectors and computing the "regular" scalar product.

Thus, the product (8.15) of a FAM with itself is the sum of squares of all its components except the local-denominated central bank liabilities. This is the same as saying that this scalar product is zero exactly on W.

Now consider the quotient space V/W, that is the space of equivalence classes, where two vectors in V are considered equivalent if they differ only by an element of W (that includes zero). Thus if v_1 and v_2 are two FAMs, and we say $v_1 = v_2$ in V/W, this means $v_1 - v_2 = w \in W$. In our case, this means that an element of V/W is the set of all FAMs in V that differ at most in the local-denominated central bank liabilities. Any valid FAM X in V automatically has a corresponding element $\pi(X)$ in V/W, namely the set of all matrices that differ from X by at most local-denominated central bank liabilities.

As the local-denominated central bank liabilities are precisely the elements ignored by the scalar product (8.15), the latter is a well-defined, positive definite scalar product on the quotient space V/W.

If the transaction matrices form a basis of V/W, then we can use a scalar product (any well-defined scalar product on V/W, so (8.15) will do) to uniquely decompose any element of V/W into a linear combination of the transaction matrices.

Let the matrix A contain the pairwise scalar products of the transaction matrices:

$$A_{\lambda_1 \lambda_2} = \left\langle \Lambda^{\lambda_1}, \Lambda^{\lambda_2} \right\rangle \tag{8.16}$$

and B be the inverse of A

$$B = A^{-1} \tag{8.17}$$

then standard linear algebra (basis change in a linear space with a scalar product, see Appendix B) shows that if we define ψ_λ by

$$\psi_\lambda = \sum_{\lambda_1} B_{\lambda \lambda_1} \left\langle \Lambda^{\lambda_1}, D\Phi \right\rangle \tag{8.18}$$

then

$$D\Phi_{d i_1 i_2} = \sum_{\lambda} \psi_\lambda \Lambda^{\lambda}_{d i_1 i_2} \text{ as elements of } V/W \tag{8.19}$$

which is the same as saying

$$D\Phi_{d i_1 i_2} = \sum_{\lambda} \psi_\lambda \Lambda^{\lambda}_{d i_1 i_2} + \sum_{i} w_i \cdot [lic] \quad \text{for some } w_i \tag{8.20}$$

Since the first term of (8.20) has no impact on net worth (being a sum of transaction matrices), the change in net worth of institution i from (8.20) is equals w_i, and therefore $w_i = S_i$ and (8.20) is in fact the decomposition (8.12).

Note that this decomposition is computationally efficient because B is a constant matrix (it is obtained from the constant transaction matrices). Thus the only computation we need to do at each time step is to compute the scalar product of $D\Phi$ with each TM, and then multiply the resulting vector by the square matrix B.

Thus the transaction matrix decomposition is made constructive. This construction also allows us to exactly answer the question "is a given set of TM's enough? Exactly enough?". The answer is as follows: if the matrix A in (8.16) is singular and thus won't invert, the matrices are not linearly independent, and one should omit some. If A inverts, one defines ψ_λ by (8.18) and then computes the residual w from (8.20). If w is indeed in W, that is, consists only of local cash entries, then we have enough transaction matrices; if not, we need to add some more to explain the other entries in the residual.

The final question worth discussing here is: what reason we have to assume that the transaction matrices do form a basis of V/W? First of all, we have constructed the TMs according to our wishes, so we can make sure they're a basis if we want to. The question then becomes, does it *make sense* to use a set of TMs that are a basis of V/W? We would say that it definitely seems wise to use enough transaction matrices to enable us to span the changes in the financial stock composition allowed by the structure of the economy. It is not as clear whether one should use "just enough" of them, or instead use a larger, linearly dependent set. Since in the latter case one could select a basis out of the larger set and represent the rest of the TMs as linear combinations thereof, the question seems to be largely one of style. If one is building a toy model and for some reason wants to allow lots of redundant transactions, there does not seem to be an a priori reason not to. However, if one wants to start from data and have a unique decomposition, as we do, it seems wise to make the TMs be a basis, as we do. The next section describes the transaction matrices we use on our dataset.

8.6 The Transaction Matrices We Use

The transaction matrices we use are listed in Table 8.2.

8.7 Summary

This chapter has described a novel way to describe financial stock dynamics. Any financial transaction typically affects several financial stocks, making it difficult to see what transactions gave rise to the observed financial stock behavior. By using a collection of constant matrices describing the structure of the financial sector in a given country, we can decompose the changes in the financial stocks into revaluation, current transactions, and capital transactions in a computationally efficient way. Conversely, the formalism derived here allows us to directly specify the portfolio allocation behavior of the different institutions while *automatically* observing the wealth constraints of the institutions. This formalism thus greatly simplifies the specification and estimation of portfolio behavior in stock-flow consistent models.

A	Private Creation of Local Currency-Denominated Deposits	$[l:pb]-[l:pc]+[l:bg]-[l:cg]$
B	Private Purchases of Government Bonds	$-[l:pc]+[l:pg]-[l:cg]$
C	Private Creation of Forex-Denominated Deposits	$-[l:pc]+[l:bg]-[l:cg]+[\$:pb]$
D	Private Local Currency-Denominated Borrowing from Commercial Banks	$[l:pc]+[l:bp]-[l:bg]+[l:cg]$
E	Private Forex-Denominated Borrowing from Commercial Banks	$[l:pc]-[l:bg]+[l:cg]+[\$:bp]$
F	Commercial Banks' Purchases of Government Bonds	$-[l:bc]+[l:bg]-[l:cg]$
G	Commercial Banks' Borrowing from Abroad	$[\$:bw]+[\$:wb]$
H	Central Bank Exchanging Currency	$-[l:wc]-[\$:cw]$
I	Commercial Banks Borrowing in Local Currency from the Central Bank	$[l:cb]+[l:bc]$
J	Commercial Banks' Purchases of Forex from the Central Bank	$-[l:bc]+[\$:bw]-[\$:cw]$
K	Central Lending to Government	$[l:cg]+[l:gc]$
L	Central Bank Borrowing From Abroad	$[\$:cw]+[\$:wc]$
M	Government Foreign Borrowing	$[l:gc]+[\$:cw]+[\$:wg]$
N	Renormalization of Government Deposits at Commercial Banks	$[l:bc]+[l:gb]-[l:gc]$
O	Central Bank Lending to Government-Owned Enterprises	$[l:pc]+[l:cp]$

Table 8.2: *Transaction Matrices Used in this Thesis*